

Topic 12: Number Properties

for use after *Bits and Pieces III* Investigation 4

As you work with operations on rational numbers, you may notice certain characteristics that hold true. For example, you know that $7 + 3 = 10$ and $3 + 7 = 10$. The Commutative Property of Addition says that you can add two numbers in any order and the result will be the same.

The table below shows some important number properties that are true for all rational numbers.

Property	Description	Examples
Commutative Property of Addition	Changing the order of the numbers you are adding does not change the sum.	$3 + 7 = 10$ and $7 + 3 = 10$, so $3 + 7 = 7 + 3$
Commutative Property of Multiplication	Changing the order of the numbers you are multiplying does not change the product.	$4 \times 5 = 20$ and $5 \times 4 = 20$, so $4 \times 5 = 5 \times 4$
Associative Property of Addition	Changing the grouping of the numbers you are adding does not change the sum.	$2 + (6 + 1) = 9$ and $(2 + 6) + 1 = 9$, so $2 + (6 + 1) = (2 + 6) + 1$
Associative Property of Multiplication	Changing the grouping of the numbers you are multiplying does not change the product.	$3 \times (2 \times 2) = 12$ and $(3 \times 2) \times 2 = 12$, so $3 \times (2 \times 2) = (3 \times 2) \times 2$

Number properties can help you perform mental calculations quickly. For example, to simplify $13 + 19 + 7$ mentally, you might use the Commutative Property of Addition to add 13 and 7 before adding 19.

Problem 12.1

Use the properties of numbers to find the value of the variable in each equation.

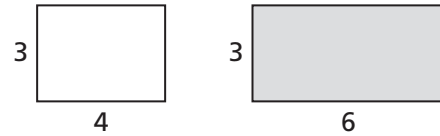
A. $3 + 9 + 12 = a + 3 + 12$

Use the Commutative Property of Addition to reorder the first two numbers, so $3 + 9 + 12 = 9 + 3 + 12$. Therefore, $a = 9$.

B. $25 \times (4 \times 12) + 3 = (4 \times k) \times 12 + 3$

Use the Associative Property of Multiplication to regroup the multiplication, so $25 \times (4 \times 12) + 3 = (25 \times 4) \times 12 + 3$. Use the Commutative Property of Multiplication to reorder the multiplication, so $(25 \times 4) \times 12 + 3 = (4 \times 25) \times 12 + 3$. So, $k = 25$.

Consider the two rectangles at the right. You can find the total area of the rectangles by writing $(3 \times 4) + (3 \times 6) = 12 + 18$, or 30.



You can also find the total area by combining the smaller rectangles into one large rectangle as shown. The rectangle area is $3(4 + 6) = 3(10)$, or 30.



This model illustrates another property of numbers. The Distributive Property combines multiplication with addition and subtraction. To multiply a sum or difference, you multiply each number within the parentheses by the number outside the parentheses. For example, you can write $3(4 + 6) = 3(4) + 3(6)$.

Problem 12.2

Use the Distributive Property to find the value of x in the equation $16(97) = 16(100) - 16(x)$.

You can write 97 as a difference of two integers, so $16(97) = 16(100 - 3)$.

Using the Distributive Property, $16(100 - 3) = 16(100) + 16(-3)$.

Since $16(100) + 16(-3) = 16(100) - 16(3)$, the complete equation is $16(97) = 16(100) - 16(3)$.

Therefore, $x = 3$.

Exercises

Name the property illustrated in each equation.

1. $15 + 27 + 25 = 27 + 15 + 25$
2. $14 \times 20 - 14 \times 5 = 14 \times 15$
3. $(5 \times 7) + (3 \times 2) = (7 \times 5) + (3 \times 2)$
4. $9 + (11 + 6) = (9 + 11) + 6$

Use mental math and number properties to simplify each expression.

5. 25×102
6. $(51 + 13) + (9 + 7)$
7. $4 \times (17 \times 25)$
8. $(20 \times 19) \times 5$
9. Given that $212 \times 4 + 7 = 855$, simplify $4 \times 212 + 7$.
10. Draw a model to show that $6(7) = 6(5) + 6(2)$.
11. Find values of x , y , and z that satisfy the equation $14(9 + 11) \times 25 + 18 \times 13 = 13 \times x + 14 \times y(11 + z)$

Topic 10: Number Properties

PACING 1 day

Mathematical Goals

- Use a factor tree to find a prime factorization
- Write a prime factorization using exponents

Teaching Guide

As students work with rational numbers, they may notice that addition, subtraction, multiplication, and division follow certain rules. These rules are described by the properties of rational numbers. Make sure that students understand that these properties hold true for all rational numbers. Some students may be confused by the use of the word “property” rather than “rule.” Students should recognize that these properties are characteristics of operations with rational numbers. Students can then use the properties to simplify mental calculations.

At this point, students are not expected to prove that these properties are true, but they should be able to use examples to demonstrate the properties. For example, you may want students to evaluate several expressions involving the Associative Property of Addition to convince them that the property is true.

Before Example 10.1, make sure that students are comfortable recognizing the properties in simple equations like $3 + 5 = 5 + 3$ and $(7 \times 6) \times 4 = 7 \times (6 \times 4)$. Students may be confused when variables or extra terms, like $+ 12$, are added to the equations.

During Example 10.1, ask:

- *Is the order of the terms different on the two sides of the equation?*
- *Is the grouping of the terms different on the two sides of the equation?*

For Exercises 5–8, students will use the properties of numbers to simplify mental calculations. Explain to students that rearranging the order of calculations or breaking a large number into two smaller numbers can help them do calculations more quickly. By using the properties of numbers, students can be sure that they are not changing the value of the expression when they simplify it.

Homework Check

When reviewing Exercise 5, ask:

- *What two numbers add up to 102 and are easy to multiply 25 by?*

When reviewing Exercise 6, ask:

- *What pairs of digits add up to 10?*
- *Which pairs of numbers in the expression add up to a multiple of 10?*
- *Which property can you use to change the grouping of the numbers in the expression?*
- *Which property can you use to change the order of the numbers in the expression?*

Vocabulary

- Associative Property of Addition
- Associative Property of Multiplication
- Commutative Property of Addition
- Commutative Property of Multiplication
- Distributive Property

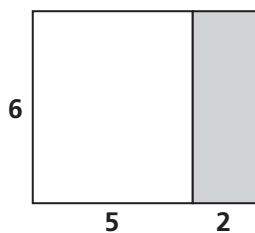
Assignment Guide for Topic 10

Core 1–11

Answers to Topic 10

Exercises

1. Commutative Property of Addition
2. Distributive Property
3. Commutative Property of Multiplication
4. Associative Property of Addition
5. 2,550
6. 80
7. 1,700
8. 1,900
9. 855
10. Answers may vary. Sample:



11. $x = 18, y = 25, z = 9$